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14. ABSTRACT

New nonlinear filtering algorithms were developed and are currently being tested. Numerical results suggest that nonlinear filters behave better than the ensemble Kalman filter methods with strongly nonlinear systems. They also seem to respect the dynamical balance of the system state more resulting in more stable predictions.

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PARTICLE KALMAN FILTERING FOR OCEAN STATE ESTIMATION

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1. LONG-TERM GOALS

The long-term scientific objective is to develop fully nonlinear Bayesian filters that generalize the optimality of ensemble Kalman filter methods to nonlinear systems and can be suitable for large dimensional data assimilation problems. The new filters are expected to perform better than the ensemble Kalman filter (EnKF) methods with comparable implementation cost. The filters will be used for realistic ocean analysis and prediction problems.

2. OBJECTIVES

Our goal is to explore new directions that would allow the implementation of the nonlinear Bayesian filtering theory with highly nonlinear systems at reasonable computational cost. We aim at developing, implementing and testing new nonlinear filters with realistic ocean data assimilation problems in mind. Simple nonlinear dynamical models will be first considered to better understand the behavior of these new filters and assess their efficiency compared to existing EnKF methods.

3. APPROACH

The solution of the nonlinear data assimilation problem can be determined from the nonlinear Bayesian filter. The filter provides the conditional probability distribution function (pdf) of the system state given all available measurements. Knowledge of the state pdf allows determining different estimates of the system state, as the minimum variance estimate. The optimal nonlinear filter recursively operates as a succession of a correction (or analysis) step at measurement times to correct the state (predictive) pdf using the Bayes' rule, and a prediction step to propagate the state (analysis) pdf to the time of the next available observation. Despite its simple algorithm, the numerical implementation of the optimal nonlinear filter can be computationally prohibitive, even for systems with very few dimensions.

The particle filter (PF) is a discrete approximation of the nonlinear Bayesian filter and is based on point-mass representation (mixture of Dirac distributions), called particles, of the state pdf. In this filter, the particles are integrated forward with the numerical model to propagate the state predictive pdf in time, and their assigned weights are updated every time new observations are available. In practice, the PF suffers from the degeneracy phenomenon where most weights become concentrated on very few particles and hence

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only a tiny fraction of the ensemble contributes to the average, causing very often the divergence of the filter. The use of more particles helps alleviating this problem over short time periods only, and the most efficient way to get around it is resampling. Besides being computationally demanding, resampling introduces Monte Carlo fluctuations which can degrade the filter's performance. Additionally, even with resampling, a very large number of particles is still required to accurately describe the continuous pdf of the system state, a necessary condition to ensure a good behavior of the filter. This makes brute-force implementation of the PF problematic with computationally demanding ocean models.

The Kalman filter (KF) provides the minimum variance solution of the data assimilation problem only when the system is linear and the statistics of the system errors are Gaussian. The Ensemble Kalman filter (EnKF) combines good properties of the PF and the linear Kalman filter. More precisely, it has the same nonlinear prediction step as the PF, but retains the "linearity aspect" of the Kalman filter in the analysis in that it applies the Kalman correction step to each particle. This means that an EnKF only updates the first two moments of the particles ensemble, and is thus semi-optimal for non-Gaussian (nonlinear) systems. Despite being "semi-optimal", many recent studies found that the EnKF is more robust than the PF when small-size ensembles are used because the Kalman-type correction of the particles reduces the risk of ensemble degeneracy by pulling the particles toward the true state of the system.

We propose to use mixture of Gaussian distributions as discrete representation of the pdf of the system state in the nonlinear Bayesian filter. A local linearization about each particle would then lead to a Kalman-type correction step for each particle complementing the usual particle-type correction. The resulting filter, referred to as Particle Kalman filter (PKF) basically runs a weighted ensemble of KFs. As in the EnKF, the Kalman-type correction step attenuates the degeneracy of the ensemble, which would allow the filter to efficiently operate with small-size ensembles. The PKF is computationally prohibitive for realistic oceanic data assimilation problems. Approaches to alleviate the computational burden of the PKF will be proposed and tested. The basic idea is to represent the pdf of the system state given the observations by mixture of Gaussian distributions with low-rank covariance matrices to derive fully nonlinear low-rank filters suitable for realistic ocean data assimilation problems.

4. WORK COMPLETED

An approach to use the optimal nonlinear filtering theory was developed for data assimilation into realistic ocean models. Different low-rank Gaussian filters were proposed and implemented, and tested with the strongly nonlinear Lorenz-96 model. It was further found that this approach sets a theoretical framework for the stochastic and deterministic ensemble Kalman filters. More precisely, the ensemble Kalman filter (EnKF) and the square-root ensemble Kalman filters (SR-EnKFs) can be derived as simplified variants from this approach. The EnKF integrates a simplified form of the PKF state pdf while the square-root filters are Gaussian-based filters. Numerical applications were performed to study the filter's behavior and evaluate their performances. The new filters were evaluated

with than the standard ensemble Kalman filters (the stochastic ensemble Kalman filter - EnKF, and the deterministic ensemble transform Kalman filter - ETKF). It was found that these new nonlinear filters work more efficiently with strongly nonlinear models providing more accurate estimates of the system states. These results were recently submitted for publication in Monthly Weather Review.

We also recently investigated the Lorenz-86 model to compare the performance of the ensemble Kalman Filter (EnKF) and the nonlinear filters. This model admits a chaotic vortical mode coupled to a comparatively fast gravity wave mode. The goal was to evaluate the performances of linear and nonlinear filters with different systems modes. To further assess the efficiency of the nonlinear analysis step in enhancing the dynamical balance of the filters solution, identical twin assimilation experiments were designed such that the true state is balanced, but the observational errors project onto all degrees of freedom, including the fast modes. It was found that EnKFs and nonlinear filters capture the variables in the slow manifold well since, once the variables are attracted towards the slow manifold, they stay there. Nonlinear filter captures slaved modes much better, implying the more robustness of nonlinear filters in handling nonlinear jumps in dependent variables. This also suggests that the solution of the nonlinear filters respects the dynamical balance of the system more. A paper discussing these results is under preparation.

5. RESULTS

New nonlinear filtering algorithms were developed and are currently being tested. Numerical results suggest that nonlinear filters behave better than the ensemble Kalman filter methods with strongly nonlinear systems. They also seem to respect the dynamical balance of the system state more resulting in more stable predictions.
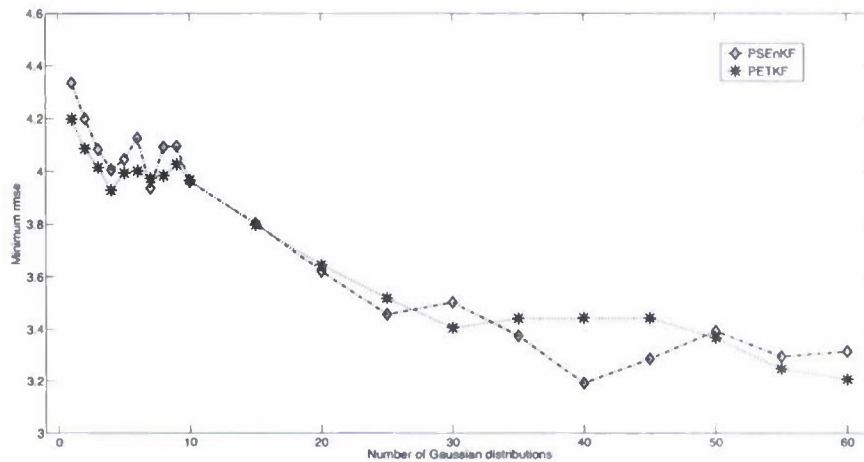
FIGURE 1.   Minimum rms errors $\hat{e}_{min}$ (over 20 experiments) of the sto-
chastic based ensemble particle filter (PSEnKF) and deterministic ensemble
transform based particle Kalman filter (PETKF) with the Lorenz 96-model
and nonlinear observations and a fixed number of 10 members in each en-
semble filter as functions of the number of Gaussian *pdf*s in the mixture. Not
that for the EnKF and ETKF correspond to the case with one component
in the Gaussian mixture used to approximate the full pdf of the stat.

6. IMPACT/APPLICATIONS

This study led to new sequential data assimilation schemes that generalizes the optimal-
ity of the ensemble Kalman filter to nonlinear systems. The new filters can be in principle
used to assimilate data to highly nonlinear ocean analysis and prediction problems. More
work is needed in this direction.

7. TRANSITIONS

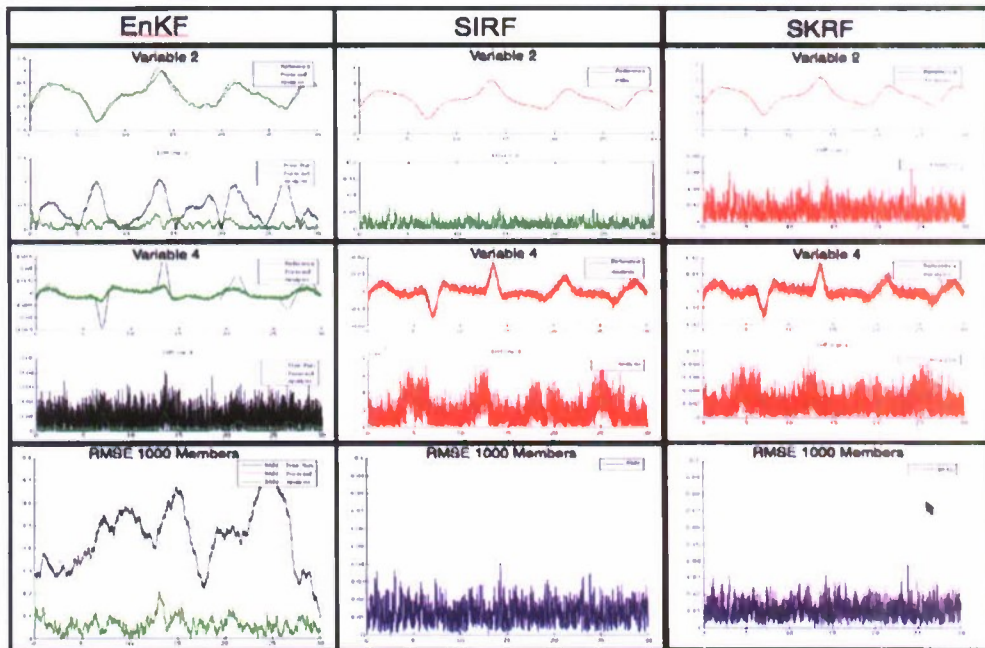Theory and algorithms can be made available to Navy scientists.

FIGURE 2. Slaved Relations Mode: Every variable at all time steps is observed. In these runs we can see that sudden jumps in the fast variable (Variable 4) are not captured well with the EnKF but the nonlinear filters tend to capture these sudden transitions better implying that nonlinear filters show a better skill in estimating nonlinear regime shifts better.
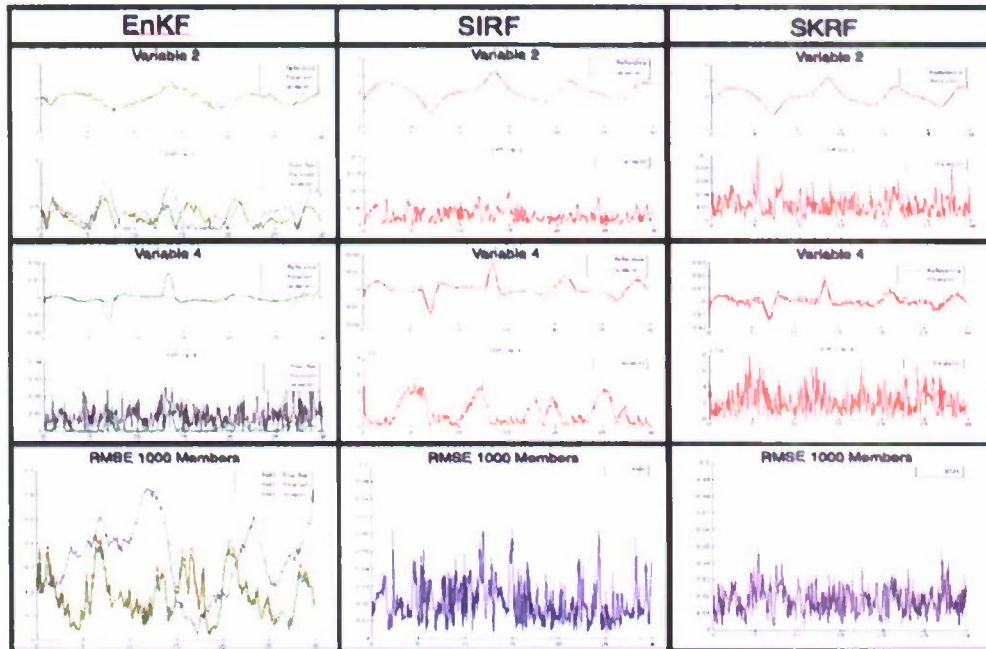
FIGURE 3. Slaved Relations Mode: First and third variable at every 8th time step is observed.In these runs we can see that sudden jumps in the fast variable (Variable 4) are not captured well with the EnKF but the nonlinears tend to capture these sudden transitions better implying that nonlinear analysis step is needed to obtain better skill in estimating nonlinear regime shifts better even with lesser number of observations.